Category theory, linguistics and pronouns

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PhD coursework examination
Paper I: Linguistic analysis
Term-paper submission 2
June 12, 2023 (v0.1)

Abstract

This article first discusses the essentials of determining a linguistic category and then we examine whether pronouns can be called a category or not. I propose that pronouns do stand as a separate linguistic category because of the features they show. This doesn't imply that every language will have them, but at least for the languages where they are found, they do demonstrate formal features of a mathematical category.

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1 Introduction

As Cheng (2015) points out, "mathematics exists to make difficult things easy, and category theory exists to make difficult mathematics easy". In mathematics, one abstracts out things which are observed in the real life in order to process the things around us faster, but the operations we perform for this also involve patterns which give rise to categories which are one step above the abstracted reality.

Exactly like in mathematics, I observe two levels of abstraction in linguistics. Initially, while collecting data, linguists abstract a lot. The production of language ('performance' in Chomskyan terms) is comprised of an enormous amount of variation which is generally ignored by linguists while profiling a language. This itself is the first level of abstraction in any linguistic research. E.g., while describing the phonology of English, even if is often noted that the /t/ sound is phonemic, the nature of that sound in a word like 'tooth' is different from, say, 'table'. Both of them are still proposed to be an instance of the same phoneme any linguistic description of English. All such sets of data are essentially abstract, since the data collectors deem a lot of actual details unnecessary for the concern at their hand.

On top of this level comes another layer of abstraction where linguists analyse the collected data and group it into several linguistic categories, e.g., nouns, verbs, adjectives. We will now look at some key-points regarding mathematical categories and then examine whether pronouns demand such a category or not.

2 Mathematical categories

Suppose we have a function f that operates on set X and results in set Y, i.e., $X = \{1, 2, 3\}$ and $Y = \{4, 5, 6\}$; function f will do $x + 3$[1] and will result in all the elements of Y. Now suppose we have $Z = \{7, 8, 9\}$, g can operate on Y and result in Z by doing $y + 3$ which itself means $(x + 3) + 3$.

A function typically takes an input called 'domain' and results in an output called 'codomain' in maths. Instead of domain and codomain, we will stick to input and output since they sound simpler (at least to me!).

Now since a clear map is observed between these functions, it can be said that $h = g \circ f$, i.e., function h is a 'composite' of functions g and f which means the output of f can be passed to g to have the resultant effect the same as having passed the same initial input to h in the first place. E.g., in our data $f(1)$ results in 4 (since $f : x + 3$). Now if we do $g(4)$, we will get 7. This two-step derivation can be reduced to a one-step function called h where $h(1)$ will directly result in 7. This results in a 'map' of functions which can lead to objects from one set to the other. The path/process from one set to the other is known as a 'morphism' in maths. By definition categories should have such morphisms.

[1]Notice the subtle difference in the notation where small letters stand for the objects from a particular set and capital letters for the set itself. So x is any object from set X.

As Lane (1971) notes, there are different kinds of morphisms possible in categories based on which the types of categories may differ. One feature of compositional morphisms is associativity which is formalised as follows:

$$a \circ (b \circ c) = (a \circ b) \circ c$$

This means that if a morphism is compositional, it will necessarily have the aforementioned equality, where the direction is important, but the order in which these functions are clubbed together, is not very important. E.g., suppose $a(x) = 2x$, $b(x) = 3x$ and $c(x) = 4x$. Giving inputs to all the functions and checking whether the equality holds true or not can be done like the following[2]:

$$\begin{aligned} a \circ (b(c(x))) &= a(b(x)) \circ c \\ a \circ b(4x) &= a(3x) \circ c \\ a(12x) &= c(6x) \\ 24x &= 24x \end{aligned}$$

3 Linguistic categories

As discussed before, linguistic analysis is also built up on an abstracted data-sets. When patterns are observed in such a data, linguists call them categories, but rarely people discuss the formal aspects of them. Let's first have a look at how formal characteristics of categories can be seen in linguistic categories.

Let's take the example of the word '*undressed*'. The '*un-*' morpheme which we will call f function, negates its input, whereas the '*-ed*' morpheme which we will call g function, changes the tense of the input to the past value[3]. Now consider the following examples:

1. He undressed himself twice during the checkup.
2. He came here and undressed.

In the first example, the meaning of him being dressed before and removing it is more prominent and on the contrary in the second one 'undressing' itself is perceived as an independent action instead of being just a reversal of some other action. Figure 1 suggests the composition of these two slightly different senses.

This reveals that these linguistic functions aren't associative. There is a high chance of ending up with different meanings if the proper order of combining isn't followed.

[2] The discussions with Sahil Patel were very helpful for understanding the concept of associativity. (Patel, Sahil; p.c. 2023-06-12)

[3] In English, the '*ed*' morpheme can also result in words which behave like adjectives, but for now we are limiting our scope for it to only denote past tense.

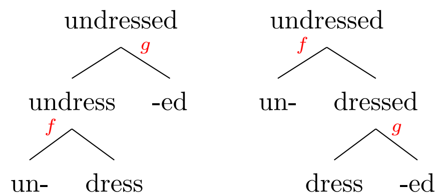

Figure 1: Compositionality in linguistic functions

Although these functions aren't associative, we sure see morphisms. In the same way how we developed functions f and g for mathematical data, we can do it for this small linguistic data. This leads us to believe that categories in linguistics can be proposed exactly like in mathematics and morphisms for input-output can be established.

4 Pronouns

Pronouns do show distinct functions than various other types of words. They belong to a closed class, but they still rely a lot on the context. Pronouns have deictic as well as referential functions which are very special functions because they interact with the system of grammar from outside it. The agent of the sentence 'I ate an apple.' cannot be determined out of context.

Another intricate difference between pronouns and other word categories can be seen in the transformative linguistic functions. A noun can be converted to a verb (cf. cash → encash), a verb to an adjective (cf. promise → promising), an adjective to a verb (cf. red → redden). If these are established as categories, these functions can be called morphisms, but categories like pronouns show no such exchange with other linguistic categories resulting in a lack of morphisms. If there aren't such morphisms, then can it really be called a category?

One important morphism which we haven't discussed so far is that of 'identity'. Only empty categories (categories having no objects) have no morphisms, otherwise objects of all other categories at least have the identity morphism. All the objects of the category of pronouns map to themselves only. This is called a 'discrete category'.

It needs more probing to claim whether referential realisations of pronouns can be thought of as morphisms or not. We surely observe mapping of pronouns with some other categories that are present in the discourse, but in order to formalise this we need a serious understanding of the linguistic exchange itself. What counts as a morphism and what doesn't will then depend upon whether the determined feature of linguistic exchange is observed in the concerned context or not. Right now, I lack such a theoretical ground and hence I can't formalise these morphisms.

5 Conclusion

I conclude that formally pronouns belong to a discrete category where all its objects are mapped to themselves with identity morphism. There is no other formal morphism observed in pronouns which maps to a different category. Pronouns have referential realisations where they definitely get mapped with other categories, but formalising them as morphisms requires formalisation of many other linguistic aspects which I haven't done in this paper, but I believe it to be a very fertile area for future research.

References

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