15.3 Cumulative Distribution Function for Chi-Square Probability Distribution

A. Purpose

The procedure described in this Section computes the Cumulative Distribution Function (CDF) of the Chi-Square probability distribution. The CDF is sometimes called the lower tail. The lower tail, or CDF, $P(\chi^2|\nu)$, and the upper tail, $Q(\chi^2|\nu)$ for the Chi-Square probability distribution with argument χ and ν degrees of freedom are defined by

$$P(\chi^2|\nu) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \int_0^{\chi^2} t^{(\nu/2)-1} e^{-t/2} dt,$$

$$Q(\chi^2|\nu) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} \int_{\chi^2}^{\infty} t^{(\nu/2)-1} e^{-t/2} dt \ = 1 - P(\chi^2|\nu)$$

B. Usage

B.1 Program Prototype, Single Precision

REAL CHISQ, NU, P, Q

INTEGER IERR

Assign values to CHISQ and NU, and obtain P = $P(\chi^2|\nu)$ and Q = $Q(\chi^2|\nu)$ by using

CALL SCDCHI (CHISQ, NU, P, Q, IERR)

B.2 Argument Definitions

CHISQ [in] Argument χ^2 of the functions $P(\chi^2|\nu)$ and $Q(\chi^2|\nu)$. Must be nonnegative. Must be positive if NU = 0.

NU [in] Degrees of freedom ν of the functions $P(\chi^2|\nu)$ and $Q(\chi^2|\nu)$. Must be nonnegative. Must be positive if CHISQ = 0.

P [out] The value of the function $P(\chi^2|\nu)$.

Q [out] The value of the function $Q(\chi^2|\nu)$.

IERR [out] A flag that normally is zero to indicate successful computation. See Section E below for discussion of non-zero values.

B.3 Modifications for Double Precision

For double precision computation, change the REAL type statement to DOUBLE PRECISION and change the initial letter of the procedure name to D.

C. Example and Remarks

See DRDCDCHI and ODDCDCHI for an example of the usage of this subprogram.

The procedures SGAMIK and SGAMIE, described in Chapter 2.19, are used to control the procedure SGAMI and determine the error estimate it returns. SGAMI is used as described in Section D below.

D. Functional Description

D.1 Method

The identities $P(\chi^2|\nu) = P(a,x)$ and $Q(\chi^2|\nu) = Q(a,x)$, with $a = \nu/2$ and $x = \chi^2/2$, where $P(a,x)$ and $Q(a,x)$ are incomplete gamma function ratios, are used. The procedure SGAMI described in Chapter 2.19 is used to evaluate $P(a,x)$ and $Q(a,x)$.

D.2 Accuracy Tests

See Section 2.19.D.

E. Error Procedures and Restrictions

The procedure SGAMI issues error messages, under several conditions, at level 2 + MSGOFF, where MSGOFF is zero unless specified by a call to SGAMIK (see Chapter 2.19) at some time before calling SCDCHI. If error termination is suppressed by setting MSGOFF < 0, or by calling ERMSET (see Chapter 19.2), IERR will be set to a non-zero value.

If the desired tolerance could not be achieved, IERR is set to 2.

If SCDCHI is called with both CHISQ and NU zero, IERR is set to 3 and P is set to 3.0.

If SCDCHI is called with one or both of CHISQ and NU negative, IERR is set to 4 and P is set to 4.0.

F. Supporting Information

Entry	Required Files
DCDCHI	AMACH, DCDCHI, DCSEVL, DERF, DERM1, DERV1, DGAM1, DGAMMA, DINITS, DRCOMP, DREXP, DRLOG, DXPARG, ERFIN, ERMSG, IERM1, IERV1
SCDCHI	AMACH, ERFIN, ERMSG, IERM1, IERV1, SCDCHI, SCSEVL, SERF, SERM1, SERV1, SGAM1, SGAMMA, SINITS, SRCOMP, SREXP, SRLOG, SXPARG

Designed and programmed by W. V. Snyder, JPL, 1993.

DRDCDCHI

```
      program DRDCHI
c>> 2001-05-25 DRDCDCHI Krogh Minor change for making .f90 version.
c>> 1996-06-17 DRDCDCHI Krogh  Minor format change for C conversion.
c>> 1996-05-28 DRDCDCHI Krogh  Changed Fortran 90 code.
c>> 1994-10-19 DRDCDCHI Krogh  Changes to use M77CON
c>> 1994-07-06 DRDCDCHI WVS set up for chgtyp
c
c     Evaluate the Probability Integral Q(chi-square,nu) of the Chi-
c     Square distribution by using DCDCHI.
c
c--D replaces "?": DR?CHI, DR?CDCHI, ?cdchi
      double precision CHISQ, NU, P, Q(4)
      integer IERR, J
c
10    format ('      Probability Integral Q(chi**2 | nu)'/
     1 '    CHI**2         NU = 1',13x,'2',13x,'3',13x,'4')
20    format (1p,5g14.7)
      chisq = 0.5d0
      print 10
30    if (chisq .le. 6) then
         do 40 j = 1, 4
            nu = j
            call dcdchi (chisq, nu, p, q(j), ierr)
40       continue
         print 20, chisq, q
         chisq = chisq + 0.5d0
         go to 30
      end if
      stop
      end
```

ODDCDCHI

```
     Probability Integral Q(chi**2 | nu)
   CHI**2         NU = 1              2              3              4
 0.5000000      0.4795001      0.7788008      0.9188914      0.9735010
  1.000000      0.3173105      0.6065307      0.8012520      0.9097960
  1.500000      0.2206714      0.4723666      0.6822703      0.8266415
  2.000000      0.1572992      0.3678794      0.5724067      0.7357589
  2.500000      0.1138463      0.2865048      0.4752911      0.6446358
  3.000000      8.3264517E-02  0.2231302      0.3916252      0.5578254
  3.500000      6.1368829E-02  0.1737739      0.3207621      0.4778783
  4.000000      4.5500264E-02  0.1353353      0.2614641      0.4060058
  4.500000      3.3894854E-02  0.1053992      0.2122903      0.3425475
  5.000000      2.5347319E-02  8.2084999E-02  0.1717971      0.2872975
  5.500000      1.9016474E-02  6.3927861E-02  0.1386386      0.2397295
  6.000000      1.4305878E-02  4.9787068E-02  0.1116102      0.1991483
```